ACKNOWLEDGMENT

The author wishes to thank Prof. N. Kumagai of Osaka University for his encouragement and K. Kitayama of Nippon Telegraph and Telephone Public Corporation for providing the experimental data and valuable discussions.

REFERENCES

[1] D. Marcuse, "Calculation of bandwidth from index profiles of optical fibers. 1: Theory," *J. Appl. Opt.*, vol. 18, no. 12, p. 2073, June 1979.
[2] K. Okamoto, "Comparison of calculated and measured impulse responses of optical fibers," *J. Appl. Opt.*, vol. 18, no. 13, p. 2199, July 1979.
[3] K. M. Case, "On wave propagation in inhomogenious media," *J. Math. Phys.*, vol 13, no. 3, p. 360, Mar. 1972.
[4] K. Morishita, Y. Kondoh, and N. Kumagai, "On the accuracy of scalar approximation technique in optical fiber analysis," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, p. 33, Jan. 1980.
[5] K. Morishita, Y. Obata, and N. Kumagai, "Accuracy of the group velocity of the step-index fiber modes evaluated by the scalar approximation technique," *Trans. Inst. Electron. Commun. Eng. Japan*, vol. 64-B, no. 1, p 87, Jan. 1981.
[6] K. Kitayama, private communication.
[7] K. Kitayama, M. Tateda, S. Seikai, and N. Uchida, "Determination of mode power distribution in a parabolic-index optical fibers: Theory and application," *IEEE J. Quantum Electron.*, vol. QE-15, p. 1161, Oct. 1979.
[8] I. H. Malitson, "Interspecimen comparison of the refractive index of fused silica," *J. Opt. Soc. Amer.*, vol. 55, no. 10, p. 1205, Oct. 1965.
[9] H. M. Presby and I. P. Kaminow, "Binary silica optical fibers: refractive index and profile dispersion measurements," *J. Appl. Opt.*, vol. 15, no. 12, p. 3029, Dec. 1979.

Characteristics of Unilateral Fin-Line Structures with Arbitrarily Located Slots

L. P. SCHMIDT, TATSUO ITOH, SENIOR MEMBER, IEEE, AND HOLGER HOFMANN

***Abstract*—Generalized unilateral fin-line configurations for extended millimeter-wave applications are analyzed using the equivalent transmission-line concept in the spectral domain. Numerical results for the frequency-dependent propagation constants and characteristic impedances of various structures are presented.**

I. INTRODUCTION

FIN-LINE STRUCTURES have proved to be a useful tool for the development of integrated millimeter-wave components (e.g., [1]). Conventional fin-line structures proposed to date are the unilateral, bilateral, and the antipodal fin-line [2], all of which are symmetric with respect to the E-plane of the shielding waveguide.

In order to improve the flexibility of this class of waveguiding structures and, thus, extending the range of application and increasing the possible degree of integration, this paper analyzes more general types of unilateral fin-lines with up to three slots in symmetric as well as asymmetric positions (Fig. 1(a)).

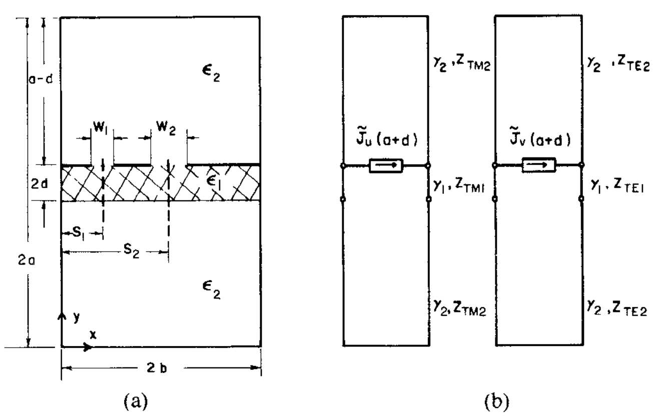

Fig. 1. Generalized unilateral fin-line (a) cross section, (b) equivalent transmission lines for TM-to-y and TE-to-y waves

This analysis includes the solution of the eigenvalue problem yielding the frequency-dependent propagation constants as well as the calculation of carefully defined characteristic impedances. Numerical results will show the improved flexibility that can be achieved by making use of this extended class of fin-line structures.

Manuscript received May 14, 1980; revised December 1, 1980. This work was supported in part by the Army Research Office under Grant DAAG29-78-G-0145. A condensed version of this paper was presented at the 1980 IEEE-MTT-S International Microwave Symposium.

L. P. Schmidt was with the Department of Electrical Engineering, The University of Texas, Austin. He is now with AEG-Telefunken, Hochfrequenztechnik, D-7900, Ulm, West Germany.

T. Itoh is with the Department of Electrical Engineering, The University of Texas, Austin.

H. Hofmann is with AEG-Telefunken, Flensburg, West Germany.

0018-9480/81/0400-0352$00.75

II. Theory

A. Formulation of the Eigenvalue Problem

Even more than a conventional spectral-domain analysis, the equivalent transmission-line concept in the spectral domain, as introduced earlier [3], is ideally suited for the derivation of matrix eigenvalue equations for planar or quasi-planar waveguiding structures. Though leading to the same results, the latter method has the advantage of being carried out in a much simpler way, and, therefore, was chosen to analyze the generalized unilateral fin-line structures.

Since in the absence of any metallization on the surfaces of the fin-line substrate the modal spectrum consists of TM-to-y and TE-to-y modes only, the modes of the structure being studied are superimposed from these fields. This is done by introducing the Fourier transform for the y-field components

$$\tilde{E}_y(\alpha, y)=\int_{-\infty}^{+\infty} E_y(x, y)\cdot e^{j\alpha x}\, dx \tag{1}$$

and similarly for $\tilde{H}_y(\alpha, y)$, where α takes discrete values

$$\alpha=n\cdot\pi/(2b), \qquad n=0, \pm 1, \pm 2, \pm 3, \cdots.$$

The remaining field components can be derived from Maxwell's equations. From the inverse transform

$$E_y(x, y)\cdot e^{-j\beta z}=\frac{1}{2\pi}\int_{-\infty}^{+\infty} \tilde{E}_y(\alpha, y)\cdot e^{-j(\alpha x+\beta z)}\cdot d\alpha \tag{2}$$

it can be seen that the fields are superpositions of inhomogeneous plane waves propagating in the $(\alpha x+\beta z)$ direction. Taking this fact into account, we transform the (x, z) into (u, v) coordinates, u being adjusted to the direction of propagation and v and y being transverse to it. Now the plane waves in the u direction are decomposed into TM-to-y $(\tilde{E}_y, \tilde{E}_u, \tilde{H}_v)$ and TE-to-y $(\tilde{H}_y, \tilde{E}_v, \tilde{H}_u)$ fields. Returning to the originally considered structure, the partwise metallization in the $(y=a+2d)$ plane is taken into account by introducing current density components $\tilde{J}_u, \tilde{J}_v$, the first generating TM, and the second generating TE fields only. The wave impedances in the y direction can be defined as $Z_{\text{TM}i}=\tilde{E}_u/\tilde{H}_v=\gamma_i/j\omega\epsilon_0\epsilon_i$ and $Z_{\text{TE}i}=\tilde{E}_v/\tilde{H}_u=j\omega\mu/\gamma_i$, where $\gamma_i=\sqrt{\alpha^2+\beta^2-\epsilon_i k^2}$ is the propagation constant in the ith region.

Up to this point, all considerations are quite general and have only been illustrated by our special example. For the formulation of the eigenvalue problem, only a few steps have to be accomplished. First, equivalent transmission lines for the TE and the TM fields are introduced, as shown in Fig. 1(b). Using simple transmission-line theory, the following relations between the "voltages" and "currents" can easily be derived:

$$\tilde{J}_u(\alpha, a+2d)=Y_{11}^e\cdot\tilde{E}_u(\alpha, a+2d) \tag{3}$$

$$\tilde{J}_v(\alpha, a+2d)=Y_{11}^h\cdot\tilde{E}_v(\alpha, a+2d). \tag{4}$$

Returning to (x, z) coordinates, we obtain the equations

$$\tilde{J}_x=\left(N_x^2\cdot Y_{11}^e+N_z^2\cdot Y_{11}^h\right)\tilde{E}_x+N_xN_z\left(-Y_{11}^e+Y_{11}^h\right)\tilde{E}_z \tag{5}$$

$$\tilde{J}_z=N_xN_z\left(-Y_{11}^e+Y_{11}^h\right)\tilde{E}_x+\left(N_z^2\cdot Y_{11}^e+N_x^2\cdot Y_{11}^h\right)\tilde{E}_z \tag{6}$$

where $N_x=\alpha/\sqrt{\alpha^2+\beta^2}$, $N_z=\beta/\sqrt{\alpha^2+\beta^2}$.

These equations relate the strip current densities and the tangential electric slot field components to each other. The expansion of the slot field components into series, and the application of Galerkin's method in the spectral domain finally lead to a homogeneous eigenvalue matrix equation, from which nontrivial solutions are derived by searching for the zeros of the matrix determinant [4].

B. Calculation of the Characteristic Impedance

The definition of characteristic impedances for modes other than genuine TEM is, to a certain degree, an arbitrary matter. Nevertheless, a carefully defined characteristic impedance is, in addition to the propagation constant, a valuable design support for practical applications.

Following the argument of Jansen in [5], we used, for the slotline type fin-line, an impedance definition via the slot voltage and the power associated with that slot

$$Z_{ci}=\frac{V_{xi}^2}{2P_i}. \tag{7}$$

This definition even holds for coupled or uncoupled slots which are asymmetrically located. The slot voltage of the ith slot can directly be found by integrating the corresponding slot field series expansion

$$V_{xi}=\int_{s_i} E_{xi}(x, a+2d)\cdot dx \tag{8}$$

whereas, the calculation of the power

$$P_i=R_e\int_0^{2a}\int_0^{2B}\left(E_{xi}H_y^*-E_{yi}H_x^*\right)dy\, dx \tag{9}$$

may be performed in the spectral domain, after Parseval's theorem has been applied to (9)

$$P_i=\frac{1}{8b}\text{Re}\sum_{n=-\infty}^{+\infty}\int_0^{2a}\left(\tilde{E}_{xi}\tilde{H}_y^*-\tilde{E}_{yi}\tilde{H}_x^*\right)dy. \tag{10}$$

By transforming the transverse-field components $\tilde{E}_x$, $\tilde{E}_y$, $\tilde{H}_x$, $\tilde{H}_y$ to those transverse components $\tilde{E}_u$, $\tilde{H}_v$, $\tilde{E}_v$, $\tilde{H}_u$ that occurred as voltages and currents in the equivalent transmission lines, the equivalent transmission line concept is introduced again. Since the y dependence of these quantities is known, the integration can be accomplished. A final transform to the field amplitudes of the expanded $\tilde{E}_x$, $\tilde{E}_z$ components yields, in combination with the slot voltage, the desired characteristic impedance.

III. Numerical Results

For a highly efficient numerical evaluation of the eigenvalue and impedance calculation, it is important to use well-suited expansion functions for the series expansion of the slot field. Therefore, sinusoidal functions, corrected by an "edge condition" term were used for the expansion (ith

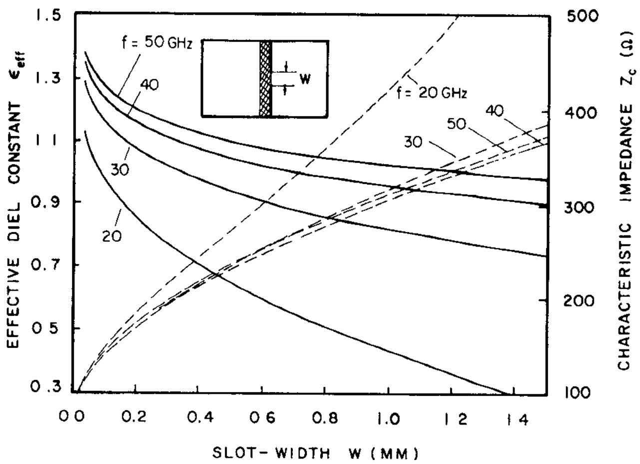

Fig. 2. Slot-width dependence of the transmission-line parameters for a single slot in central position. Identical quantities for Fig. 2–Fig. 6: f=33 GHz; $2a$=7.112 mm; $2b$=3.556 mm; $2d$=0.125 mm; ϵ_r=2.2. ——: effective dielectric constant $\epsilon_{\rm eff}$; - - - - -: characteristic impedance Z_c.

slot: w_i = slot width, s_i = slot center coordinate)

$$\{E_{xi}(x), E_{zi}(x)\} = \left\{ \sum_m a_{xim} \cdot f_{xim}(x), \sum_m a_{zim} \cdot f_{zim}(x) \right\}$$

$$\{f_{xim}, f_{zim}\} = \frac{\{\cos, \sin\}\left[m\pi(x - s_i - w_i/2)/w_i\right]}{\sqrt{1 - \left[2(x - s_i)/w_i\right]^2}}, \quad \text{for } s_i - w_i/2 < x < s_i + w_i/2$$

$$\{f_{xim}, f_{zim}\} = \{0,0\}, \qquad \text{otherwise.}$$

These functions are readily Fourier transformed analytically.

Convergence checks with up to five expansion functions for each component show that for reasonable slot widths, a 0.5-percent accuracy for both the propagation constant and the impedance can be achieved by using only two elements of the expansion.

The fin-line structures analyzed with the present method include configurations of up to three slots in symmetric as well as asymmetric positions. In all cases, the fin-line was composed from a WR-28 waveguide and a substrate with a dielectric constant of 2.2 and a thickness of 0.125 mm.

Fig. 2 shows the slot-width dependence of both the effective dielectric constant $\epsilon_{\rm eff} = \beta^2/k^2$, and the characteristic impedance Z_c of the dominant mode for a single slot in symmetric position for several frequencies. The frequency behavior of Z_c agrees well with the bilateral case as investigated in [6] for a wide frequency range. In this symmetric case, only even spectral terms (n even) and even expansion functions (m even) need to be considered for the computation of the dominant mode, of course.

In Fig. 3, a single slot is continuously shifted away from the symmetric position towards one of the sidewalls. This results in only a small change of $\epsilon_{\rm eff}$, whereas the impedance change, especially for broader slots, is more significant.

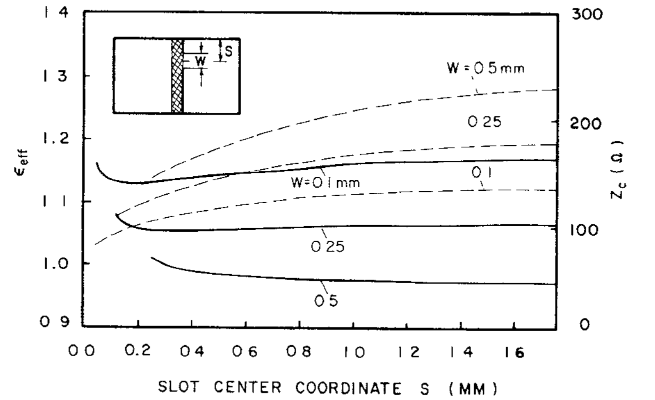

Fig. 3. Characteristics of a single slot shifted away from the central position towards one of the sidewalls.

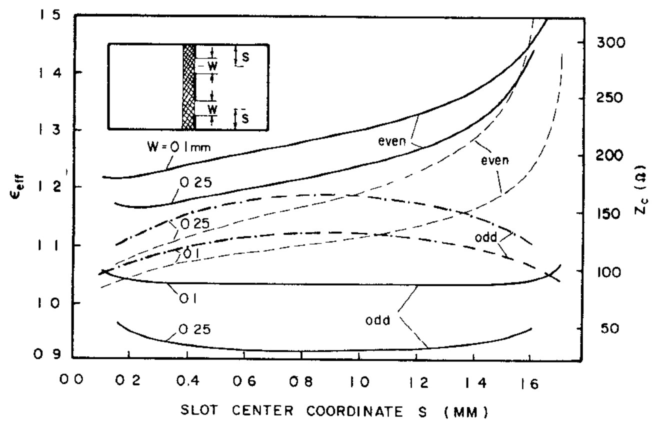

Fig. 4. Two symmetrically positioned slots. Abscissa is their distance from the sidewalls.

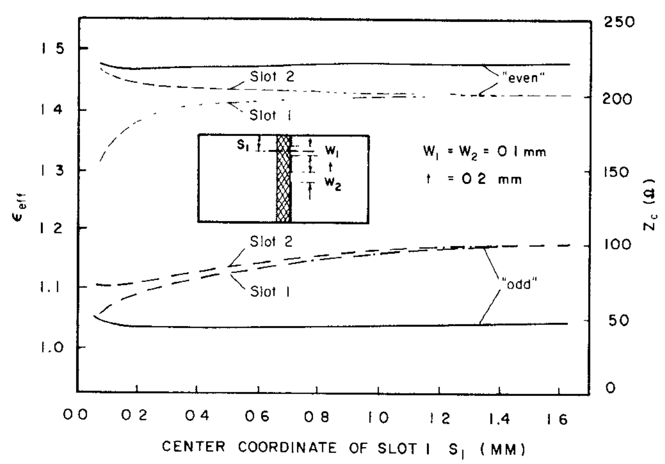

Fig. 5. Shift of two strongly coupled slots from a centered position towards a sidewall.

The characteristic of two coupled slots in symmetric positions with varying distance from each other is shown in Fig. 4. The "symmetric" behavior of the odd mode with respect to $s = b/2$ becomes intelligible if a (not disturbing) electric wall is added in the symmetry plane $x = b$. In fact, the characteristic of the odd mode was confirmed by taking only half of the waveguide with one asymmetric slot into account. Especially for strongly coupled slots, both $\epsilon_{\rm eff}$ and

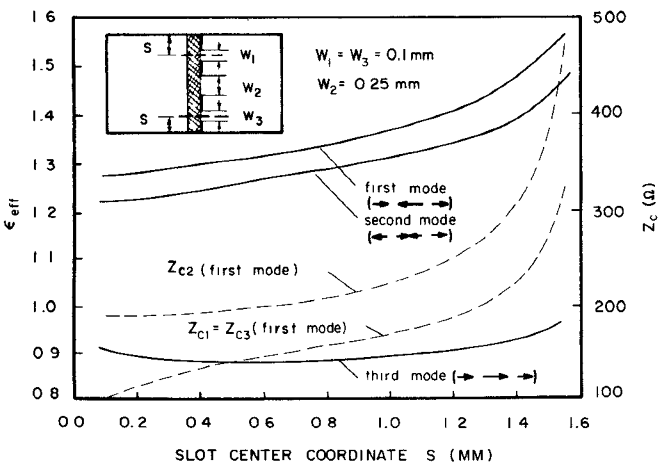

Fig. 6. Unilateral fin-line with three slots, one in central position, being approached from both sides by smaller slots.

Z_c of the even mode are very sensitive to a change of the slot distance.

The influence of a shift of two such strongly coupled slots out of a centered position is only small in the beginning, as shown in Fig. 5. This is a consequence of the high concentration in the slot region. The characteristic impedances of the two slots, of course, obtain different values now, as one slot is closer to the approached sidewall than the other.

In the last example, Fig. 6, the influence of two narrow slots, approaching a broader slot in symmetric position, is investigated. The figure includes the propagation constants of the three propagating modes as well as the impedances of the first and most important mode. Symmetrically tapered transitions from the one- to three-slot case probably excite only this first mode so that the line parameters of the centered slot can be tuned without changing the width or the position of this slot.

IV. Conclusions

New unilateral fin-line configurations have been proposed that provide a wider range of flexibility for the development of integrated millimeter-wave components. The method presented includes the calculation of both the frequency-dependent propagation constant and a carefully defined characteristic impedance, and can be applied to bilateral symmetric or asymmetric fin-line structures as well.

References

[1] H. Hofmann, H. Meinel, and B. Adelseck, "Integration of millimeter wave components (in German)," *Nachrichtentech. Z* [*Commun. J.*], vol. 31, pp. 752–757, 1978.

[2] H. Hofmann, "Dispersion of planar waveguides for millimeter-wave application," *Arch. Elek. Übertragung* [*Electron. Commun.*], vol. 31, pp. 40–44, 1977.

[3] T. Itoh, "Spectral domain immittance approach for dispersion characteristics of shielded microstrips with tuning septums," in *Conf. Dig. E. M. C.*, Brighton, England, 1979.

[4] ____, "Analysis of microstrip resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 946–952, 1974.

[5] R. Jansen, "Unified user-oriented computation of shielded, covered and open planar microwave and millimeter-wave transmission-line characteristics," *Microwaves Opt. Acoust.*, vol. MOA 1, pp. 14–22, 1979.

[6] L. P. Schmidt, and T. Itoh, "Spectral domain analysis of domonant and higher order modes in fin-lines," *IEEE Trans. Microwave Theory Tech.*, to be published.